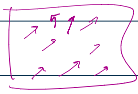
Math 53, Discussion 105

Monday, April 6, 2020 9:02 AM

Chapter 16.1 (vector fields), Chapter 16.2 (line integrals)

A vector field on R^2 is: A map $\mathbb{R}^2 \to \mathbb{R}^2$

$\mathbf{F}(x,y)$ 2-vector Like wind directions on a plane



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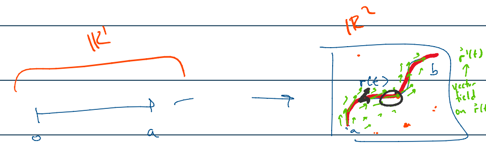


$\vec{r}(t)$ is a curve in $\mathbb{R}^2$

$\mathbb{R} \to \mathbb{R}^2$

The gradient of a function is: A vector field

$\nabla f(x,y)$ is a 2-vector for each x,y

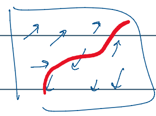
$F(t)$ $\mathbb{R}^2 \to \mathbb{R}^2$ is a vector field

A conservative vector field is:

$\to F = \frac{d}{dx} f$

a vector field $F(x,y)$ s.t. $\boxed{F(x,y) = \nabla f(x,y)}$

Scalar function



$\vec{r}(t)$

To integrate a scalar function along a curve C we write:

$\mathbb{R}^2 \to \mathbb{R}$

$r_1(t) = (t,t)$ $0 \le t \le 1$

a curve C can be parametrized

$r_2(t) = (1-t, 1-t)$

$t=0$ to $t=1$

$$\int_C f(x,y)\,ds = \int_a^b f(x(t),y(t))\sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\,dt$$

as $\vec{r}(t) = (x(t), y(t))$

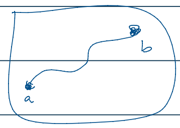
for $a \le t \le b$

$(\cos(t), \sin(t))$

$(\sin(t), \cos(t))$

plug in your parametrization

speed of curve



To integrate a vector field along a curve C we have:

$\mathbb{R}^3 \to \mathbb{R}^3$

Param. by $\vec{r} : \mathbb{R}^1 \to \mathbb{R}^3$

$$\boxed{\int_C F \cdot dr} = \int_a^b F(\vec{r}(t)) \cdot r'(t)\,dt = \int_C F \cdot T\,ds = \int_C P\,dx + \overset{Q}{P}\,dy + \overset{R}{P}\,dz$$

Where $F = \langle P, Q, R \rangle$

$\vec{r}\,'(t) = \frac{d}{dt}\vec{r}(t)$

$\vec{r}\,'(t)\,dt = d\vec{r}(t)$

Exercises:

1)

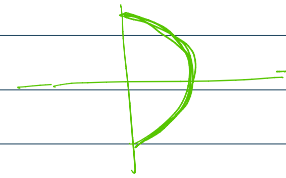
2) $\int_C xy^4\,ds$ with C the right half of the circle $x^2 + y^2 = 16$

3) $\int_C F \cdot dr$ with $F = \sin(x)\,i + \cos(y)\,j + xz\,k$ and $r = t^2 i - t^2 j + tk, 0 \le$

2) $\int_C xy^4\, ds$ with C the right half of the circle $x^2 + y^2 = 16$

3) $\int_C F \cdot dr$ with $F = \sin(x)\, i + \cos(y)\, j + xz\, k$ and $r = t^2 i - t^2 j + tk, 0 \le t \le 1$

4) Compute $\int_C (1 + xy)e^{xy} dx + (x^2 e^{xy} + x) dy$ with C the unit circle.
(Also find a $f(x, y)$ such that ∇f is the integrand).



$\int_C xy^4\, ds$ let $\vec{r}(t) = (4\cos(t), 4\sin(t))$ $-\frac{\pi}{2} \le t \le \pi/2$

$\sqrt{(x')^2 + (y')^2} = \sqrt{4^2} = 4$

$= \int_{-\pi/2}^{\pi/2} 4^5 \cos(t) \sin^4(t) \cdot 4\, dt$

$=$ Calculus

3) $\int_0^1 \underbrace{2t\sin(t^2) - 2t\cos(-t^2) + t^3}_{F(r(t)) \cdot r'(t)}\, dt$

4) $= 0$